

## COGNITIVE ILLUSIONS RECONSIDERED

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Behavioral economists have done a great service in connecting psychology and economics. Up to now, however, most have focused on cognitive illusions and anomalies, in order to prove the descriptive failure of neoclassical economic models. Some conjectured that “mental illusions should be considered the rule rather than the exception” (Thaler, 1991, p. 4), thus questioning the assumption that probabilistic judgments are consistent and unbiased. In an influential article, Rabin (1998) concluded: “Because these biases lead people to make errors when attempting to maximize  $U(x)$ , this research poses a more radical challenge to the economics model” (p. 11).

Not everything that looks like a fallacy, however, is one. Economists have been presented a lopsided view of research in psychology (e.g., by Rabin, 1998). Here we explain three of the factors producing the phenomena labeled cognitive illusions: inefficient representations (in the context of base rate fallacy), selected sampling of problems (in the context of overconfidence and availability), and narrow norms (in the context of conjunction fallacy). Understanding these factors allows us to gain theoretical insight into the processes underlying judgment and decision making and to design effective tools to help people reason under uncertainty.

We begin with the power of representations. The argument is that the human mind does not work on information, but on representations of information. Many cognitive illusions disappear if one pays attention to this fundamental property of human thinking. For instance, evolved representations of information, such as natural frequencies, promote probabilistic reasoning, whereas conditional probabilities tend to create cognitive illusions. We illustrate the power of representations by demonstrating how one can foster Bayesian reasoning in laypeople and experts.

### 1. Base Rate Fallacy Reconsidered

Optimal allocation decisions in markets involve updating probabilities. When probabilities need to be updated to reflect new information, people are assumed to reason in a Bayesian way. In other words, people are assumed to be rational Bayesian EU maximizers. But are they really? Many experimenters have concluded that people lack the

ability to make Bayesian inferences, even in simple situations involving a binary predictor and criterion: “Man is Apparently not a Conservative Bayesian: He Is not Bayesian at All” (Kahneman and Tversky, 1972, p. 450). Consider breast cancer screening with mammography, which incurs costs of about \$2 billion every year in the U.S. Given how much money is spent for this technology, physicians and patients should understand what its outcome means.

A woman tests positive and asks her physician how likely it is that she has breast cancer. The relevant statistical information for the woman’s age group is:

The probability of breast cancer is 1% [base rate]; the probability of a positive test given breast cancer is 90% [sensitivity]; and the probability of a positive test given no breast cancer is 10% [false positive rate].

The posterior probability  $p(H | D)$  that a woman who tests positive actually has breast cancer can be calculated by Bayes’ rule. Here,  $H$  stands for hypothesis, such as cancer, and  $D$  for data, such as a positive test result:

$$\begin{aligned} p(H | D) &= \frac{p(H)p(D | H)}{p(H)p(D | H) + p(\text{not-}H)p(D | \text{not-}H)} \\ &= \frac{(.01)(.90)}{(.01)(.90) + (.99)(.10)} \approx .08. \end{aligned} \quad (1)$$

That is, roughly 9 out of every 10 women who test positive do not have breast cancer. Radiologists, gynecologists, and other physicians, however, tend to overestimate this probability by an order of magnitude (Gigerenzer, 2002). For instance, some physicians believe this probability to be 90% – and for two different reasons. They either mistake the sensitivity for the posterior probability, or alternatively, they subtract the false positive rate from 100%, which results in the same estimate. In both cases, the base rate is ignored – an instance of the “base-rate fallacy.” Overestimating the chance of breast cancer after a positive screening test exacts unnecessary physical, psychological, and monetary costs. For instance, every year more than 300,000 American women who do not have breast cancer undergo a biopsy as a consequence of false positives, and for every \$100 spent on screening, an additional \$33 is spent on following up on false positive results (Gigerenzer, 2002). We will now show how to improve laypeople’s and experts’ reasoning by selecting a more efficient representation of the statistical information.

## 2. The Ecological Argument

To understand and evaluate the performance of the human mind, one needs to look at its environment, in particular at the external representation of information. Mathematical probabilities are representations of uncertainty that were first devised in the 17th century (Gigerenzer et al., 1989). For most of the time during which the human mind evolved, information was encountered in the form of natural frequencies, that is, counts

that have not been normalized with respect to base rates. Representation matters because Bayesian reasoning is relatively simple with natural frequencies, but becomes cumbersome the moment conditional probabilities (or normalized frequencies) are introduced.

An example of a representation in terms of natural frequencies is:

Ten of every 1000 women have breast cancer; 9 of those 10 women with breast cancer will test positive and 99 of the 990 women without breast cancer will also test positive.

How many of those who test positive actually have breast cancer? Natural frequencies help people to see the answer: Nine out of the 108 women who tested positive actually have cancer. Natural frequencies facilitate Bayesian computations because they carry information about base rates, whereas normalized frequencies and probabilities do not. If  $a$  is the frequency of  $D$  and  $H$  (e.g., positive test and disease), and  $b$  the frequency of  $D$  and not- $H$ , then the posterior probability can be calculated as follows:

$$p(H | D) = \frac{a}{a + b} = \frac{9}{9 + 99} \approx .08. \quad (2)$$

Note that with natural frequencies, base rates need not be directly attended to. In contrast, if natural frequencies have been normalized with respect to the base rates, resulting in conditional probabilities or relative frequencies, one has to multiply the normalized values by the base rates in order to bring the base rates “back in” [compare Equations (1) and (2)]. Unlike conditional probabilities, natural frequencies are an efficient representation for Bayesian reasoning because the representation does part of the computation.

This insight provides a powerful tool for improving probabilistic reasoning, in laypeople as well as physicians and other professionals.

### 3. Helping John Q. Public

The majority of demonstrations of the base rate fallacy involved people with no specific competence in probabilistic reasoning. In almost all of these demonstrations, they encountered the statistical information in the form of probabilities. Would natural frequencies help? As Figure 1 shows, in each of 15 problems, including the breast cancer problem, the cab problem, and other standard problems in the literature, natural frequencies increased the proportion of Bayesian inferences substantially (Gigerenzer and Hoffrage, 1995). On average, people reasoned the Bayesian way with probabilities in only about 1 out of 6 cases, whereas in 1 out of 2 cases they did so with natural frequencies.

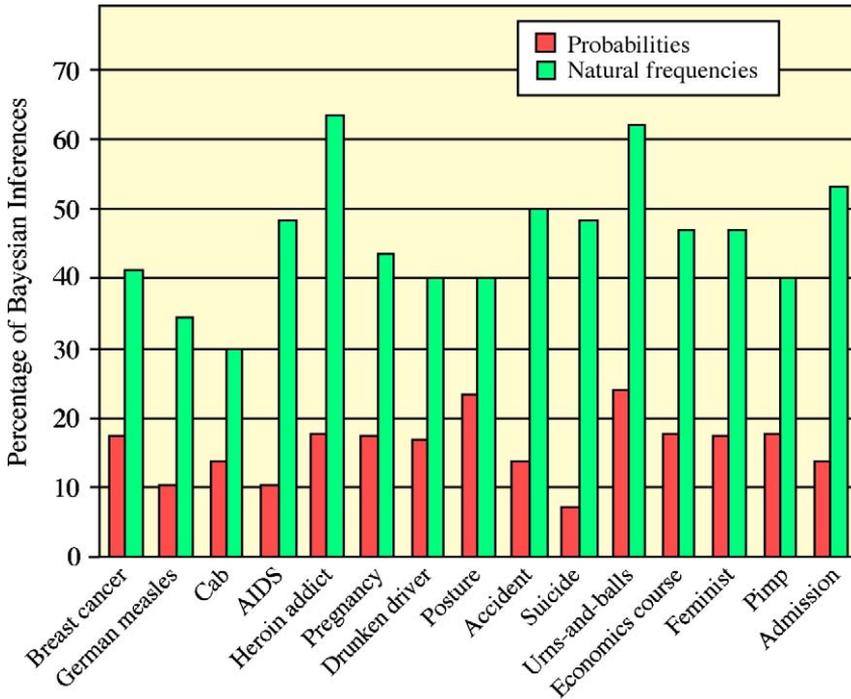


Figure 1. How an efficient representation – natural frequencies – improves probabilistic reasoning in laypeople. Sixty participants were tested on 15 problems each. They had to infer, for instance, the probability of breast cancer given a positive mammogram (see text), or the probability of severe prenatal damage of a newborn given the mother suffered from German measles during pregnancy. To qualify as a Bayesian inference, the participant had to respond with the exact Bayesian estimate, and the written protocol had to confirm that the response was derived from actual Bayesian reasoning. The statistical information was either presented in conditional probabilities or in natural frequencies. In each problem, probabilistic reasoning improved when statistical information was communicated in natural frequencies (Gigerenzer and Hoffrage, 1995).

#### 4. Helping Physicians

Would the same simple method work with physicians who make diagnostic inferences in daily practice? Figure 2 (left) shows the answers of 48 physicians to the mammography problem. When the statistical information was presented in probabilities, as it is common in medical textbooks, then their estimates of the probability that a woman has breast cancer given a positive mammogram varied between 1% and 90%. If women knew about this disturbing variability, they would be rightly alarmed. When the same information was presented in natural frequencies, the physicians’ estimates clustered around the correct answer. Figure 2 (right) shows the same positive effect of an efficient representation for colorectal cancer screening, that is, estimating the chance that a patient has colorectal cancer given a positive hemoccult test.

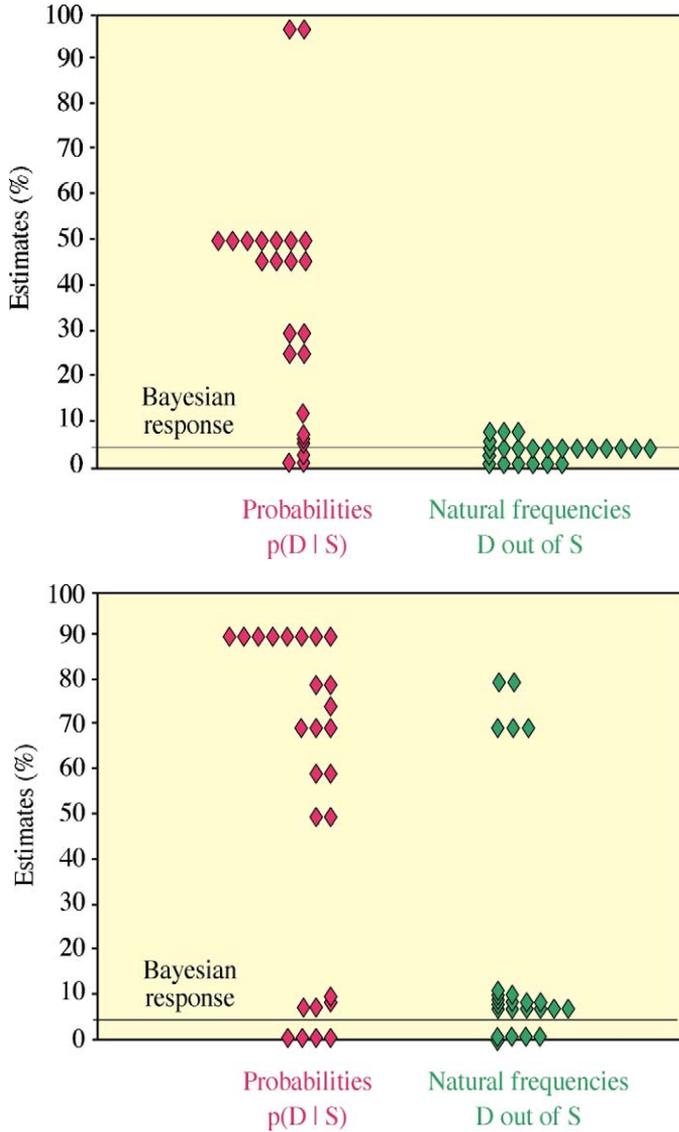


Figure 2. How an efficient representation improves probabilistic reasoning in physicians. Of 48 physicians with an average of 14 years of professional experience, half received the statistical information in conditional probabilities, the other in natural frequencies. Each point represents one physician. The ordinate shows their estimates of the probability or frequency of breast cancer (colorectal cancer) after a positive screening test. With conditional probabilities, the physicians were highly inconsistent; with natural frequencies, this inconsistency largely disappeared and the physicians' estimates clustered around the correct estimate (from Gigerenzer, 2002; Hoffrage and Gigerenzer, 1998).

## 5. Helping AIDS Counselors

Between 1986 and 1996, U.S. federal spending for AIDS research, educational programs, counselor training, testing, and prevention programs increased from \$300,000 to \$9 billion. In the U.S., some 50 million blood and plasma samples are tested every year. Most HIV tests are performed on low-risk clients, for whom the base rate of infection is very small. Though HIV tests are excellent, they are occasionally in error, which makes probabilistic thinking indispensable. Do professional AIDS counselors know what a positive test result means?

In a study by [Gigerenzer, Hoffrage, and Ebert \(1998\)](#), one of the authors went undercover to 20 public health centers to take 20 HIV tests. He used the mandatory pretest counseling session to ask questions about base rates for low-risk clients, sensitivity, false positive rate, and his chances of having HIV were he to test positive. None of the 20 counselors communicated information in natural frequencies; all used conditional probabilities and got confused without noticing. Fifteen of the 20 counselors estimated the chances that the client has HIV were he to test positive (in both the Elisa and Western blot tests) as 99.9% or higher. Natural frequencies, in contrast, help to replace confusion by insight. Out of every 10,000 men with no risky behavior, about one will be infected by HIV, and he will test positive with practical certainty. Of the other 9999, one will falsely test positive. Thus, we can expect that out of two men who test positive, only one has the virus. Again, the best technology does not suffice when people do not understand their products. In the case of HIV testing, efficient representations can help to avoid psychological and physical costs, ranging from losing one's job to contemplating suicide.

## 6. Helping Lawyers and Judges

In the counseling room as well as in the courtroom, choosing an efficient representation can make the difference between life and death. Like medical tests, DNA fingerprinting requires reasoning about base rates, false positives, and false negatives. Notwithstanding this fact, out of some 175 accredited law schools in the U.S., only one requires a course in basic statistics. [Lindsey, Hertwig, and Gigerenzer \(2003\)](#) asked advanced law students and legal professionals (including law school professors) to evaluate two criminal case files based upon two actual rape and murder cases. In both cases, a match was reported between the DNA of the defendant and a trace on the victim. When the statistical information was expressed as conditional probabilities, only 13% of the professionals and less than 1% of the law students correctly inferred the probability that the defendant was actually the source of the trace given a match ([Figure 3, left](#)). When the identical information was stated in terms of natural frequencies, the correct inferences increased to 68% and 44%, respectively. Did the representation also matter for the verdict? Yes. More professionals and students voted "guilty" when the evidence was presented in terms of probabilities, that is, when their minds were clouded ([Figure 3, right](#)).

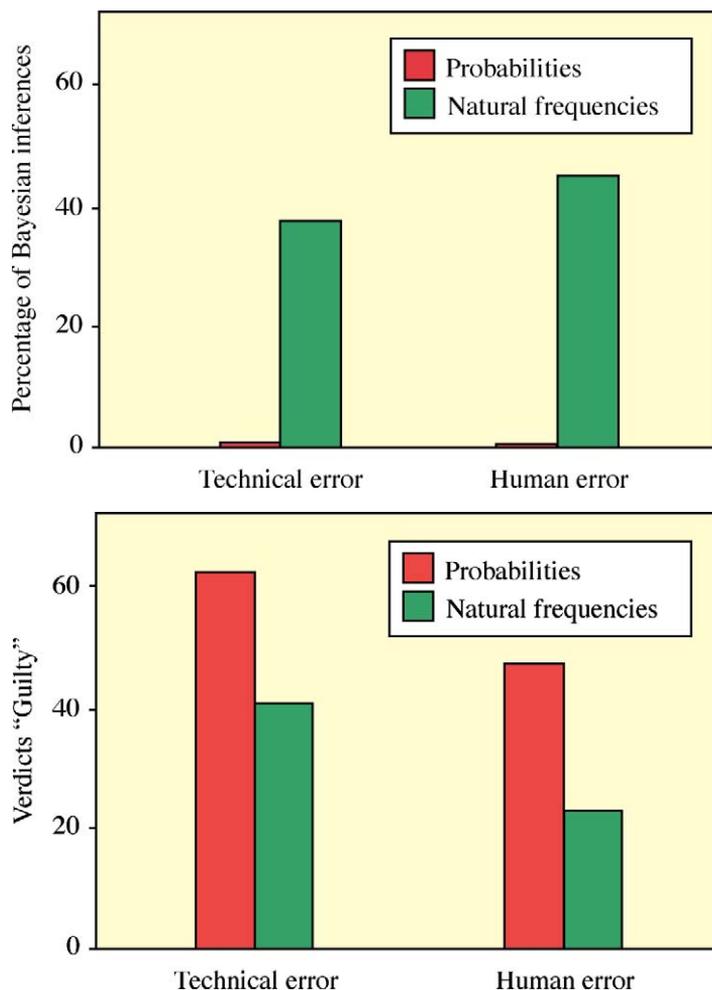


Figure 3. How an efficient representation improves probabilistic reasoning and influences verdicts in legal professionals and law students. When 27 professionals and 127 law students responded to DNA evidence presented in terms of conditional probabilities, few could correctly infer the probability that the defendant was actually the source of the trace, given a DNA match. With natural frequencies, more than 40% of the students and the majority of the professionals “saw” the correct answer (a). Representation also influenced participants’ ultimate verdict (b). With conditional probabilities, more students and professionals voted “guilty” (from Hoffrage et al., 2000; Lindsey, Hertwig, and Gigerenzer, 2003).

## 7. How to Teach Bayesian Reasoning

In the studies reported so far, probabilistic reasoning improved without instruction. The effects observed can be boosted by explicitly teaching people to translate probabilities

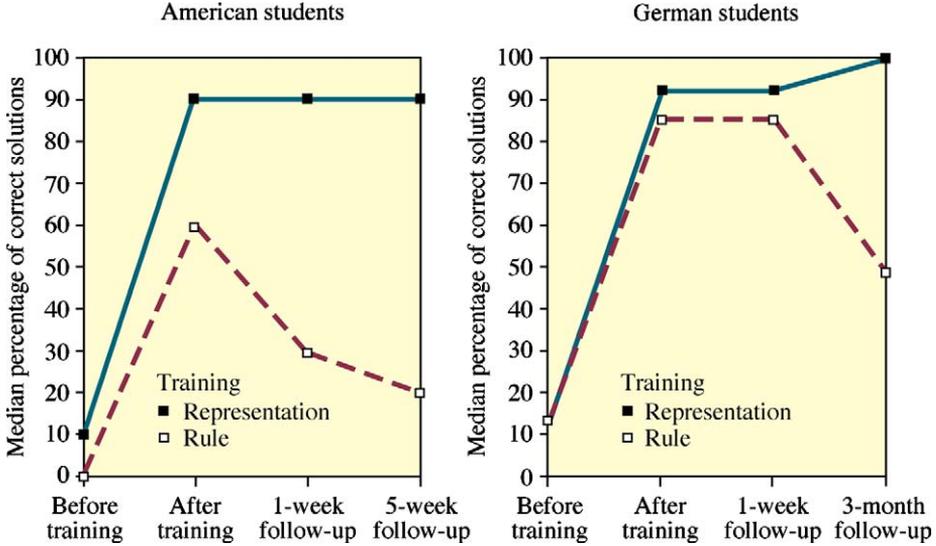


Figure 4. How to learn and not forget after the exam. With the traditional method of teaching how to insert probabilities into Bayes' rule (rule training), American and German students tend to forget what they have learned. However, when they have been taught to translate probabilities into natural frequencies (representation training), performance remains high (from Gigerenzer, 2002; Sedlmeier and Gigerenzer, 2001).

into natural frequencies. Sedlmeier and Gigerenzer (2001) (Sedlmeier, 1999) designed a tutorial computer program that teaches people to translate probabilities into natural frequencies (representation training) or, alternatively, to insert probabilities into Bayes' rule (rule training). As every teacher knows, the problem is not so much to get statistical knowledge into students' minds, but to keep it there after the exam. Figure 4 shows, that for both for American and German students, rule training leads to the typical forgetting curve, whereas representation training results in robust probabilistic thinking lasting over the entire time examined.

To conclude: The base rate fallacy, or more generally, the difficulties people have in reasoning with conditional probabilities, is often presented as if it were the natural consequence of flawed mental software. This view, however, overlooks the fundamental fact that the human mind processes information through external representations, and that how representations are selected can improve or impair our performance in statistical reasoning.

### 8. Overconfidence Bias Reconsidered

Overconfidence bias has become one of the stock-in-trade examples of research on cognitive illusions. Many kinds of economic disasters, from the large proportion of start-ups

that quickly go out of business to the exaggerated confidence of financial investors, have been attributed to this alleged cognitive illusion. “[S]ome basic tendency toward overconfidence appears to be a robust human character trait” (Shiller, 2000, p. 142). These conjectures have been justified by reference to experiments in which confidence is studied with general knowledge questions of the following kind:

Which city has more inhabitants?

(a) Hyderabad (b) Islamabad

How confident are you that your answer is correct?

50%, 60%, 70%, 80%, 90%, 100%.

The typical finding is that when people say they are 100% confident, the relative frequency of correct answers is only about 80%. When they are 90% confident, the proportion correct is about 75%, and so on. This systematic discrepancy has been interpreted as a cognitive illusion and labeled overconfidence bias (e.g., Lichtenstein, Fischhoff, and Phillips, 1982). Quantitatively, overconfidence bias is defined as the difference between mean confidence and mean percentage of correct answers. Like many other cognitive illusions, overconfidence bias has been claimed to be a stable fallacy: “...can anything be done? Not much” (Edwards and von Winterfeldt, 1986, p. 656).

We know now that much can be done. Overconfidence is nothing like a robust character trait but a consequence of three determinants that can be manipulated outside people’s minds: the question researchers ask, the sampling technique researchers employ, and the regression phenomenon. Challenging the view that “overconfidence bias” reflects a shortcoming of the human mind, Erev, Wallsten, and Budescu (1994) showed that regression to the mean is a sufficient condition for the systematic discrepancy (imperfect calibration) called overconfidence bias to arise. That is, an ideal statistical device would generate a similar discrepancy, namely estimates that regress towards the mean. Concerning the first determinants, one can ask a frequency question after a series of items: How many of the last 50 questions did you answer correctly?

If overconfidence were indeed a stable character trait, asking a frequency question rather than the typical confidence question should not affect overconfidence. Yet the question makes overconfidence disappear. Moreover, Figure 5 shows that overconfidence bias can be made to appear, disappear, or even reverse into underconfidence by using random samples of questions rather than selected samples. In random samples, pairs such as Hyderabad–Islamabad, where a cue with high ecological validity (Islamabad is a capital, and capitals tend to have a large number of inhabitants) is misleading, are no longer overrepresented. The effects of frequency questions and random sampling were first shown by Gigerenzer, Hoffrage, and Kleinbölting (1991). In a subsequent study, Griffin and Tversky (1992) replicated the effect of frequency questions, but disputed the effect of sampling. However, a meta-analysis of 135 studies finally showed that overconfidence consistently disappears when questions are randomly sampled, and that this finding cannot be attributed to a hard-easy effect (Juslin, Winman, and Olsson, 2000).

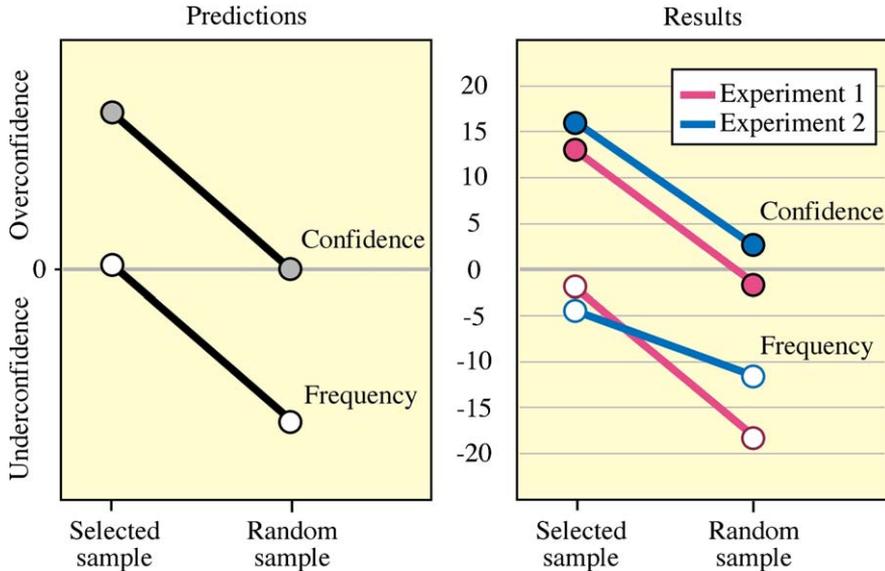


Figure 5. How to make overconfidence disappear – or even reverse to underconfidence. The predictions (left panel) are derived from the theory of probabilistic mental models (Gigerenzer, Hoffrage, and Kleinbölting, 1991), which specifies the processes underlying confidence and frequency judgments. The results of two experiments (right panel) show that when questions were randomly sampled from a natural environment (here: all German cities with more than 100,000 inhabitants), then overconfidence bias disappeared in confidence judgments. Frequency judgments (How many questions did you answer correctly?) showed no overconfidence in selected samples, and exhibited underconfidence in random samples. This simultaneous demonstration of over- and underconfidence indicates that overconfidence is nothing like a stable mental trait. Rather, this result is consistent with a heuristic process that is adapted to specific environmental structures but can be let astray in others.

To summarize, the experimental phenomenon that has been (mis)labeled overconfidence bias can be fully explained by three determinants in a person’s environment: the question researchers ask, the sampling technique researchers employ, and the regression phenomenon. There is no reason to attribute the experimental results to robust shortcomings of the human mind. To understand how overconfidence and underconfidence are generated, one has to look outside the individual mind – such as to the sampling process (Figure 5). The theory of probabilistic mental models specifies how cognitive heuristics lead to these phenomena as a function of the environments in which they operate (Gigerenzer, Hoffrage, and Kleinbölting, 1991).

## 9. Conjunction Fallacy Reconsidered

A most elementary rule of probability is the conjunction rule, which states that the joint probability  $p(A \wedge B)$  cannot exceed  $p(A)$ . “[A] system of judgments that does

not obey the conjunction rule cannot be expected to obey more complicated principles that presuppose this rule, such as Bayesian updating, external calibration, and the maximization of expected utility” (Tversky and Kahneman, 1983, p. 313). Not surprisingly, following these authors’ report that most people commit the “conjunction fallacy,” this cognitive illusion has since been invoked to explain various economic and societal problems. These include John Q. Public’s unreasonable fear of technological risks such as nuclear reactor failures (Stich, 1985), his questionable spending on insurance (Johnson et al., 1993), and even major blunders in U.S. security policy (Kanwisher, 1989).

The classic problem designed to demonstrate violations of the conjunction rule is the Linda problem (Tversky and Kahneman, 1983):

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Rank the following statements by their probability:

Linda is a bank teller (T).

Linda is active in the feminist movement (F).

Linda is a bank teller and is active in the feminist movement (T&F).

The typical result was that 80 to 90% of participants judged T&F to be more probable than T, a judgment inconsistent with the conjunction rule. This judgment was labeled the “conjunction fallacy” and soon became the primal sin of human rationality. However, one ought to be cautious of labeling these judgments a fallacy. With Tversky and Kahneman’s norm for rational reasoning, the content of the Linda problem is irrelevant; one does not even need to read the description of Linda. All that counts are the terms “probability” and “and,” which they assume must be interpreted as the mathematical probability and logical AND, respectively. However, as the Oxford English Dictionary shows, this is far from true: These words have multiple legitimate meanings in natural language; only few of them correspond to the mathematical probability and logical AND, and therefore need not obey the conjunction rule. To reveal the various ways people understand the natural language word “probability” in the Linda problem, Hertwig and Gigerenzer (1999) asked participants to paraphrase it. Bearing testimony to the polysemy of “probability,” participants responded with 18 different interpretations (Figure 6). Across all their responses, only 18% were mathematical, a number that corresponds to the usual 80 to 90% violations found.

To test this polysemy argument, Hertwig and Gigerenzer (1999) used the same description of Linda but asked participants a frequency question:

Imagine 200 women who fit the description of Linda. How many of the 200 women are:

Bank tellers (T).

Active in the feminist movement (F).

Bank tellers and are active in the feminist movement (T&F).

Figure 6 shows that the paraphrases switched towards mathematical probability, and Figure 7 shows that the violations of the conjunction rule largely disappeared when the frequency question clarified what the task is about (see also Fiedler, 1988; Mellers, Her-

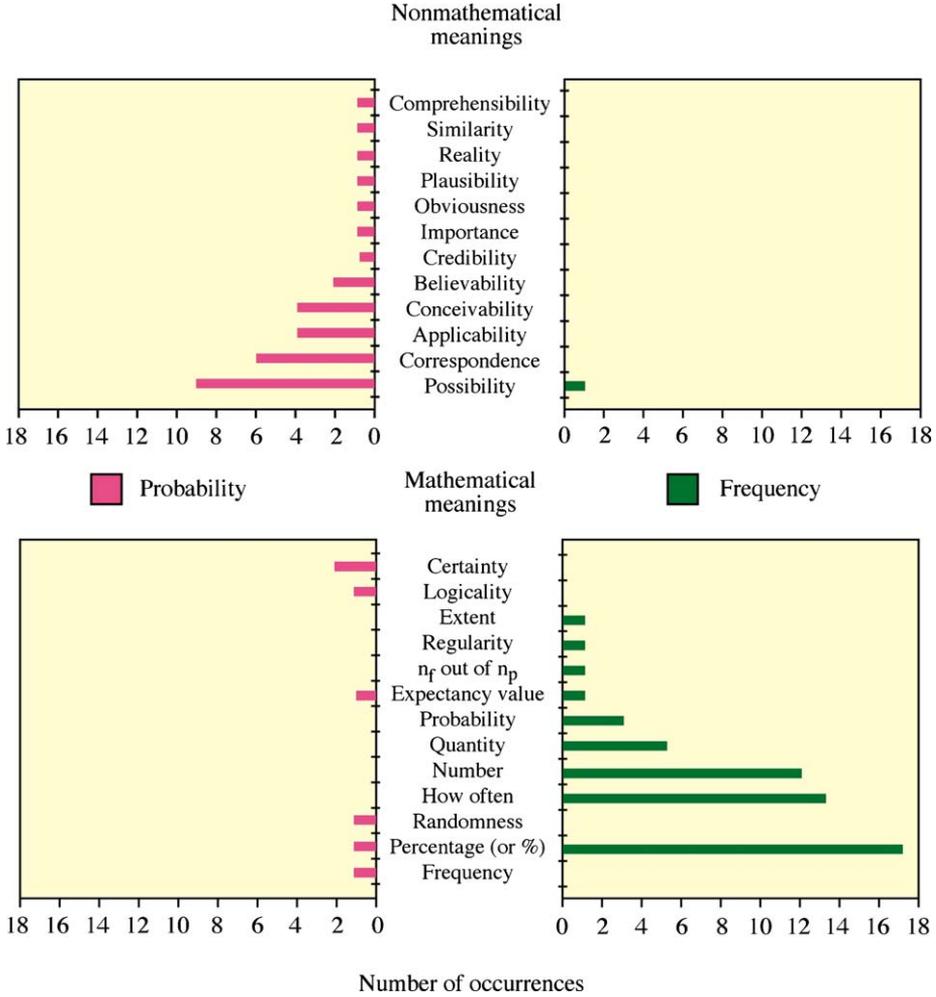


Figure 6. The word “probability” is interpreted non-mathematically and the word “frequency” is interpreted mathematically in the Linda problem, making the conjunction rule irrelevant and relevant, respectively. Participants were first presented with either the probability or the frequency version of the Linda problem. Then they were asked to imagine themselves in the role of an experimenter who must describe the Linda problem verbally to a participant who is not a native speaker, and for whom the term “probability” or “frequency” must be paraphrased. The bars show the frequency of participants’ paraphrases for the terms “probability” (red bars) and “frequency” (green bars). For details see Hertwig and Gigerenzer (1999).

twig, and Kahneman, 2001; Tversky and Kahneman, 1983). This effect is moderated by response mode (people are more likely to violate the conjunction rule when instructed to give ranks rather than estimates; Hertwig and Chase, 1998).

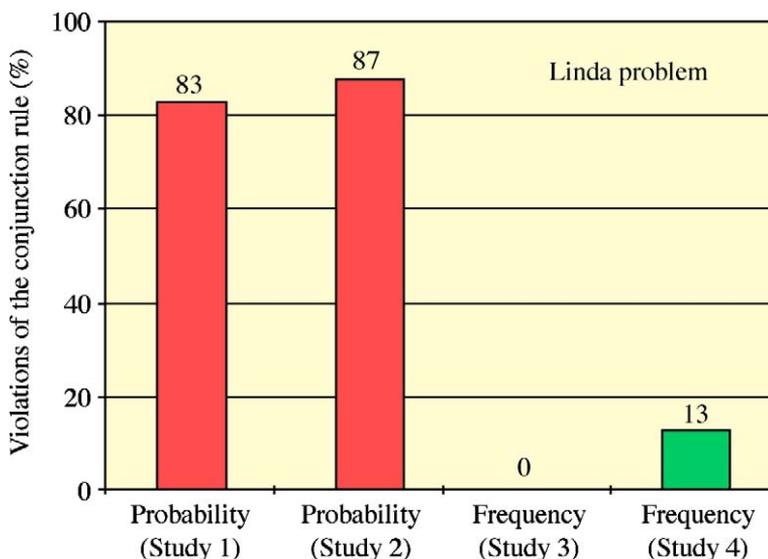


Figure 7. Frequency judgments can make the “conjunction fallacy” disappear. Hertwig and Gigerenzer (1999, Studies 1–4) asked participants to judge either the probability or the frequency of statements T, F, and T&F (see text). The bars show the percentages of violations of the conjunction rule for probabilities (red bars) and frequencies (green bars).

To summarize, when the polysemy of the English term “probability” is eliminated, the phenomenon dubbed the “conjunction fallacy” largely disappears, too. Any computer can mechanically apply the conjunction rule. In contrast, the ability to infer the meaning of polysemous terms from the context they are embedded in is an impressive human capability unmatched by any computer. In the present case, the relevant context is the fact that the experimenter provided a description of Linda, which suggests legitimate non-mathematical meanings of probability such as possibility and conceivability. By interpreting people’s semantic inferences as a conjunction fallacy, human social intelligence has been mistaken for irrationality.

## 10. Availability Reconsidered

Behavioral economics is often criticized for merely listing anomalies without providing a theory. Whether or not this is true in general, the attempt to explain human behavior in terms of “availability,” “representativeness,” and “anchoring and adjustment” is a case in point. These labels are too vague to count as explanations and, post hoc, one of them can account for almost any phenomenon. Representativeness, for instance, refers to some form of similarity judgment. However, psychological research has since long proposed and tested precise models of similarity, including Euclidean distance, City

Block distance, and likelihood ratios. In light of these models, the new preference for vague terms such as representativeness reflects a step backwards, which is in itself an interesting phenomenon. The danger is that these one-word explanations account for everything and nothing.

The term availability has been used to explain distorted frequency or probability judgments. Beginning with the original work (Tversky and Kahneman, 1973), this term has been attributed various ambiguous meanings, such as the number of instances that come to mind and the ease with which the operations of retrieval can be performed. Despite, or possibly because of its vagueness, availability has repeatedly been invoked to explain various phenomena – for instance, why the purchase of earthquake insurance rises after a quake, why personal experience distorts crime risk perception, or more generally why “people disproportionately weigh salient, memorable, or vivid evidence even when they have better sources of information” (Rabin, 1998, p. 30).

Given how little theory there is, the weight of the argument rests on the experimental evidence. In a widely cited study designed to demonstrate how people’s judgments are biased due to availability, Tversky and Kahneman (1973) had people estimate whether each of five consonants (*K, L, N, R, V*) appears more frequently in the first or the third position in English words. Each of these five selected consonants actually occurs more frequently in the third position, which is untypical because the majority of consonants occur more frequently in the first position. Thus, the test sample was deliberately unrepresentative. Two-thirds of participants judged the first position as being more likely for a majority of the five consonants. This result was interpreted as a demonstration of a cognitive illusion and attributed to the availability heuristic: Words with a particular letter in the first position come to mind more easily. While this latter assertion may be true, there was no independent measure of availability in this study, nor has there been a successful replication in the literature.

Sedlmeier, Hertwig, and Gigerenzer (1998) defined the two most common meanings of availability. Thus they were able to measure them independently of people’s frequency judgments, and test whether availability can actually predict them. The number of instances that come to mind was measured by the number of retrieved words within a constant time period (availability-by-number), and ease of retrieval was measured by the speed of the retrieval of the first word for each letter (availability-by-speed). Figure 8 shows the predictions of both versions of the availability heuristic and people’s actual estimates. The test involved a large sample of letters rather than the five consonants, which, as described above, were untypical. Neither of the two versions of availability predicted people’s actual frequency judgments. Rather, the judgments were roughly a monotonic function of the actual proportions, with a regression toward the mean, that is, an overestimation of low and an underestimation of high proportions.

To summarize, vague notions such as availability have been used as surrogates for theory. For a classic “demonstration” of availability, we showed that when one independently defines and measures it, availability does not account for people’s frequency judgments. It is understandable that when availability and similar notions were first proposed in the early 1970s, they were only loosely characterized. Yet, more than

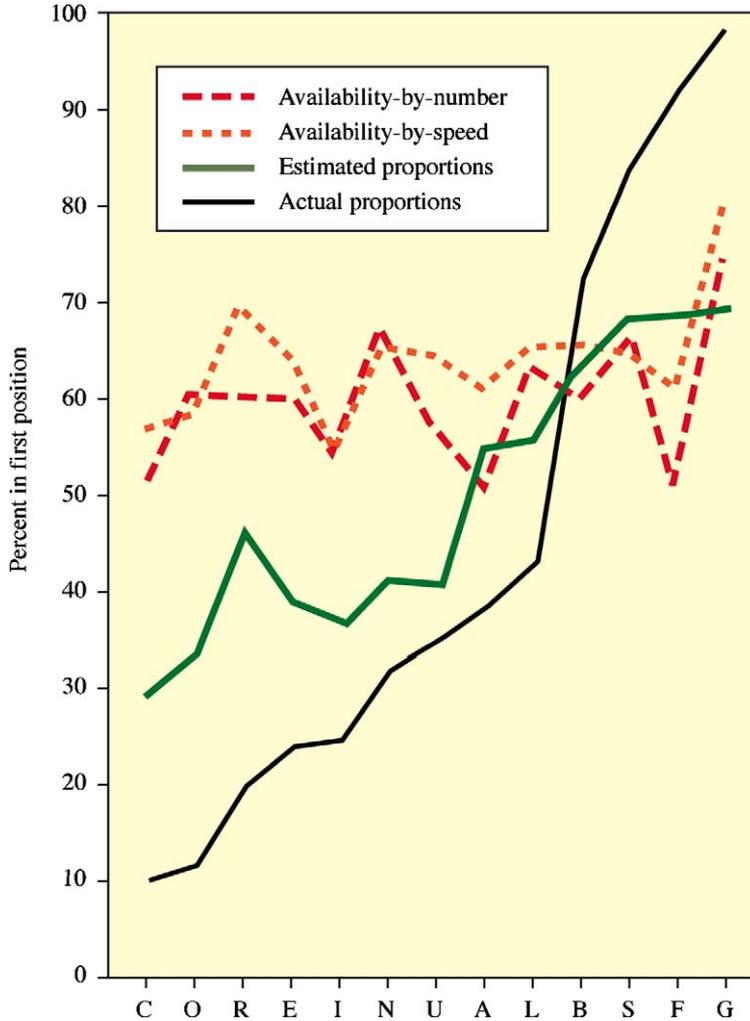


Figure 8. Participants were presented with either one or several vowels and consonants. They were asked to estimate the relative frequency with which each letter occurs in the first position compared to the second position in German. The red lines show the empirically derived predictions of two versions of the availability heuristic. The means of participants' relative frequency estimates in three subsequent studies are plotted against these predictions. The green line shows these estimated proportions transformed into percent in the first position (where the sum of the frequencies with which a particular letter occurs in the first and second position equals 100%). The black line shows the actual relative proportions with which the letters appear in the first position (calculated from an extensive German text corpus), with the rank ordered from left to right.

three decades later, the reluctance to define precise models has become a burden to the progress in connecting psychology and economics.

## 11. Conclusion

The key problems in the cognitive illusions literature can be summarized in two terms: narrow norms and vague heuristics (Gigerenzer, 1996). The fact that the cognitive illusions we have dealt with can be reduced by efficient representations, or turn out to be no illusions at all, should not lead to the conclusion that people make no errors. By definition, any intelligent system that can operate in an uncertain world will make errors. When one defines precise models of heuristics, one can predict in which tasks people who use them will fail and where they will succeed. For instance, the hindsight bias is a memory distortion that corresponds to the feeling “I knew it all along.” The “Take The Best” heuristic, as part of an adaptive memory updating mechanism that has the hindsight bias as its by-product, can predict in which task hindsight bias will and will not occur (Hoffrage, Hertwig, and Gigerenzer, 2000). The task ahead is to model the cognitive processes underlying judgment and decision making. Once we understand them, we will be able to both predict when judgments are likely to go astray and help people make reasonable decisions in an uncertain world.

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