

## Does the Environment Have the Same Structure as Bayes' Theorem?

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Cognition should not be divorced from its environment, argued Egon Brunswik (1964), comparing the two to a married couple who have to come to terms with one another by mutual adaptation. His “ratiomorphic explication” of cognition started with analyzing the statistical texture of the natural environments (the ecological validities) and the degree to which perception is adapted to that texture. Anderson’s program of “rational analysis” is quite similar: To specify the statistical structure of the environment, and, on the assumption that cognition is adapted to that structure, to infer the structure of cognition—or, at least, to infer constraints imposed on cognition by the environment. Both Brunswik and Anderson study the coming-to-terms of the married couple as an adaptation of only one partner (cognition) to the other, and both view the mind as an intuitive statistician. But here the similarities end.

*Does Anderson pursue his own program?* The crucial Step 2 of the “rational analysis” is “to specify the structure of the environment to which cognition is optimized,” which “is much easier to observe than the structure of the mind.” How do we observe that environmental structure? Among three approaches, Anderson proposes the “appeal to existing scientific theory” as the most compelling, to be illustrated with a rational analysis of categorization. So let us look at that: What is the structure of the environment that is reflected in the structure of category formation? Anderson proposes two structural components, the disjoint partitioning of the object set, and the independence of features of objects. Both are necessary assumptions for his Bayesian modelling of categorization and other cognitive functions. In the case of categorization, the evolutionary rationale Anderson gives is twofold: (1) that species cannot interbreed (disjoint partitioning of the object set), and (2) that features within species are displayed largely independently of one another.

Even if these two structural components were characteristic for the evolutionary context—Anderson himself admits that independence does not hold when features are controlled by the same gene—the question is whether they are characteristic for other contexts, too, as Anderson assumes. Conditional independence is a mathematically convenient assumption in standard Bayesian models, but not necessarily valid in natural environments. Brunswik in fact focused on the *dependencies* between features of objects in natural environments, which for him defined the texture of an environment. Similarly, physicians look for clusters of dependent symptoms to arrive at a disease classification. In general, conditional nonindependence among testimonial evidence, clinical symptoms, and other features poses a well-known problem in the sequential application of Bayes’ rule, as it does in Anderson’s “rational theory.” Dependence between the prior probability and the likelihood ratio (which measures the impact of new information) in Bayes’ theorem poses another problem (e.g., Birnbaum, 1983).

This may be sufficient to illustrate why I do not think that Anderson pursues his own program: to analyze the structure of the environment. Rather, he seems to have started with Bayes’ theorem as a model of rationality and to have assumed that the structural assumptions underly-

ing Bayes' theorem specify the structure of the environment as well. This is a legitimate heuristic: to start with some statistical model of inference—Fisher's analysis of variance, Neyman-Pearson decision theory, multiple regression, Bayes' theorem—and to investigate the hypothesis that the mind is an intuitive statistician of that kind or school (Gigerenzer, 1991). And this is what I understand Anderson to be doing. But this is not Anderson's program according to his own lights.

*Is Bayesian statistics adaptive?* “The information-processing implications of various environmental cues are not certain. This fact leads to the Bayesian character of the optimization analysis. . . .” Why? The same fact leads Brunswik to the multiple-regression character of optimization. Neither Brunswik nor Anderson explains why they believe that their respective statistics would be adaptive. Bayesianism seems to be flexible enough to apply to any environment, even to those commonly seen to contradict it, such as Allais' and Ellsberg's paradoxes (e.g., Jeffrey, 1987). But insofar as there is a specific Bayesian model of some cognitive function, I believe that the question whether the model applies to a given environment can be answered only empirically, not a priori. This can be done by checking whether a *structural isomorphism* exists between a given environment (or task) and the specific Bayesian model (see Gigerenzer & Murray, 1987, pp. 162–174). Similarly, if we want to see the mind as a rational intuitive statistician (Bayesian or otherwise), then we need to postulate not only a statistical algorithm, but in addition some heuristics (or a second-order algorithm) that check whether the structural assumptions of the algorithm hold in the given environment over time and space.

*Toward domain-specific theories of cognition.* One direction for revising the rational analysis would be to change the singular form “to specify the structure of the environment” (Step 2) into the plural form “to specify the structures of environments.” Different environments may have different structures, and these may also change over time. Thus, the program would need an extra step before Step 2 to obtain a categorization of various environments. Let us call the product of this categorization a set of domains. Domains may correspond with respect to level of abstraction and predictive power to Rosch's basic level objects (e.g., Rosch, 1978). For example, the recent proposal of domain-specific theories of reasoning has greatly advanced the potential to predict people's information search in the Wason selection task (e.g., Cheng & Holyoak, 1985; Cosmides, 1989). Proposed domains (of human interaction) include social contracts, threats, permissions, obligations. In a social contract, for example, a decisive structural component seems to be that a participant can be cheated and that subjects consistently search for information that can reveal potential cheaters (see Maynard Smith: “Game Theory and the Evolution of Behaviour”, *BBS*, 7 [1], 1984; and Caporael et al.: “Selfishness Examined: Cooperation in the Absence of Egoistic Incentives”, *BBS*, 12 [4], 1989).

Bayesian models can indeed be very useful in suggesting a conceptual language for talking about differences in structures across domains. But domains also have surplus structures, such as cheating options, which go beyond standard statistical structures. If we take Anderson's program seriously and start with a theory of environments (as opposed to starting with Bayes' theorem) then we might indeed make the “substantial discovery” that Anderson promises. But we might also discover that cognition is more flexible and does not always rely on Bayes' theorem and strong assumptions such as independence. A highly adaptive intuitive statistician of the mind might even work with exploratory data analysis.